First

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Birzeit University- Mathematics Department
Calculus II-Math 132

First Exam — Spring 2012/2013

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Instructor of Discussion(Arabic):..Dr....Ikleh.............. Section:(10)............12:00 الساعة السبت يوم

Time: 90 Minutes — **There are 4 questions in 7 pages.**

Question 1.(51%) Circle the correct answer:

1. $\sinh(\ln 2) =$

   (a) $\frac{3}{2}$.

   (b) $\frac{3}{4}$.

   (c) $\frac{5}{4}$.

   (d) $\frac{5}{2}$.

$\frac{e^x - e^{-x}}{2} = \frac{e^{\ln 2} - e^{-\ln 2}}{2} = \frac{\frac{2\times 2}{2\times 1} - \frac{1}{2}}{2} = \frac{4-1}{2}\cdot\frac{1}{2} = \frac{3}{4}$

2. To solve $\int \frac{x^3+2}{x^4-1}dx$ using partial fractions, we write

   (a) $\frac{x^3+2}{x^4-1} = \frac{Ax^4+Bx^3+Cx^2+Dx+E}{x^4-1}$.

   (b) $\frac{x^3+2}{x^4-1} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x^2+1}$.

   (c) $\frac{x^3+2}{x^4-1} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1}$.

   (d) $\frac{x^3+2}{x^4-1} = \frac{Ax^2+Bx}{x^2-1} + \frac{Cx+D}{x^2+1}$.

$\frac{x^3+2}{(x^2-1)(x^2+1)} = \frac{A}{x-1} + \frac{B}{(x+1)} + \frac{Cx+D}{(x^2+1)}$

3. $\int_0^1 xe^{2x}\,dx =$

   (a) $1+e^2$.

   (b) $\frac{1+e^2}{4}$.

   (c) $\frac{1+e^2}{2}$.

   (d) $\frac{e^2}{4}$.

x	e^{2x}
1	$\frac{e^{2x}}{2}$
0	$\frac{e^{2x}}{4}$

$u = 2x$, $du = 2\,dx$, $dx = \frac{du}{2}$

$\frac{x}{2}e^{2x} - \frac{e^{2x}}{4}$

$e^{2x}\left(\frac{x}{2} - \frac{1}{4}\right)\Big]_0^1 = e^2\left(\frac{1}{2} - \frac{1}{4}\right) - \left(1(0 - \frac{1}{4})\right)$

$e^2\left(\frac{1}{4}\right) + \frac{1}{4} = \frac{e^2+1}{4}$

4. The half-life of polonium is 139 days. The decay rate k is

   (a) $\frac{139}{2}$.

   (b) $\frac{2}{139}$.

   (c) $\frac{139}{\ln 2}$.

   (d) $\frac{\ln 2}{139}$.

$t_{\frac{1}{2}} = \frac{\ln 2}{k}$

$\frac{\ln 2}{139}$

5. One of the following statements is <u>false</u>

$\frac{e^x - e^{-x} + e^x + e^{-x}}{2}$

(a) $\sinh x + \cosh x = e^x$.

(b) The range of $\sinh x$ is $(-\infty, \infty)$.

(c) $\cosh 0 = 1$.

(d) $\frac{d}{dx}(\text{sech}x^2) = 2x(\tanh x)\text{sech}x$. X

$-\text{sech}\, x^2 (\tanh x^2) \cdot 2x$

6. $\int_1^e \frac{2^{\ln x}}{x} dx$ $= \int \frac{2^u}{x} \cdot du$ $\int 2^u du$

$u = \ln x$

$du = \frac{1}{x} dx$

$dx = du\, x$

(a) $\frac{1}{\ln 2}$.

(b) $\ln 2$.

(c) 2.

(d) 1.

$= \frac{2^u}{\ln 2}\Big|$

$= \frac{2^{\ln x}}{\ln 2}\Big|_1^e = \frac{2^{\ln e}}{\ln 2} - \left(\frac{1}{\ln 2}\right) = \frac{2-1}{\ln 2} = \frac{1}{\ln 2}$

7. If $f'(x) = \tan x$, the length of the curve $f(x)$, $0 \le x \le \frac{\pi}{4}$ is

$L = \int_0^{\pi/4} \sqrt{1 + (f')^2}$

(a) $1 + \ln(\sqrt{2})$.

(b) $\ln 2$.

(c) $\ln(\sqrt{2} + 1)$.

(d) $\ln \sqrt{2}$.

$\int_0^{\pi/4} \sqrt{1 + \tan^2} = \int_0^{\pi/4} \sec x = \ln|\sec + \tan|\Big|_0^{\pi/4}$

$= \ln|\sqrt{2} + 1| - (\ln 1 - 0)$

8. Using the substitution $x = \sin\theta$, we can write $\int \frac{\sqrt{1-x^2}}{x^2} dx$ as

$dx = \cos\theta$

$= \int \frac{\sqrt{1 - \sin^2\theta}}{\sin^2\theta} = \int \frac{\cos\theta}{\sin^2\theta} \cdot \cos\theta$

(a) $\int \csc\theta \, d\theta$.

(b) $\int \cot\theta \csc\theta \, d\theta$.

(c) $\int \csc^2\theta \, d\theta$.

(d) $\int \cot^2\theta \, d\theta$.

$\int \frac{\cos^2\theta}{\sin^2\theta}$

$\cancel{= \int \frac{\cos\theta \csc\theta \, d\theta}{\sin\theta}}$

$= \int \cot^2\theta$

9. The area of the surface generated by revolving the line $y = x$, $0 \le x \le 1$ about the y-axis is

$\int 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$ $\frac{dx}{dy} = 1$

$= \int_0^1 2\pi y \sqrt{1 -}$

(a) $\sqrt{2}\pi$.

(b) 2.

(c) $\sqrt{2}$.

(d) 2π.

10. $\int_1^e \ln\sqrt{x}\,dx =$

(a) $e-1$.

(b) $\ln(1+e)$.

(c) $\frac{1}{2}$.

(d) 1.

$u = \ln\sqrt{x}$ $\quad dv = dx$

$du = \frac{1}{2\sqrt{x}} \cdot \frac{1}{\sqrt{x}}\,dx \quad v = x$

$= x\ln\sqrt{x} - \int x \frac{1}{2x}\,dx = x\ln\sqrt{x} - \frac{1}{2}x \Big|_1^e$

$= \left(e\ln e^{\frac{1}{2}} - \frac{1}{2}e\right) - \left(0 - \frac{1}{2}\right)$

$= e\ln e^{\frac{1}{2}} - \frac{1}{2}e + \frac{1}{2}$

$\frac{e}{2} - \frac{e}{2} + \frac{1}{2}$

11. $\int_0^{\pi/4} \tan^3\theta\,d\theta =$ $\quad \int_0^{\frac{\pi}{4}} \tan\theta(\sec^2\theta - 1)$

(a) $\frac{1}{2} + \ln\left(\frac{1}{\sqrt{2}}\right)$. $\quad = \int \tan\theta\sec^2\theta - \int \tan\theta$

(b) $\frac{1}{2} + \ln\sqrt{2}$.

(c) $1 + \ln\sqrt{2}$.

(d) $\frac{\pi}{2}$.

$u = \tan\theta$

$du = \sec^2\theta$

$d\theta = \frac{du}{\sec^2\theta}$

$\sec^2\theta - \tan^2\theta = 1$

$= \int u\sec^2\theta \cdot \frac{du}{\sec^2\theta}$

$\frac{u^2}{2} = \left(\frac{\tan^2}{2} - \ln|\cos\theta|\right)\Big|_0^{\frac{\pi}{4}}$

$= \frac{1}{2} - (-\ln\frac{1}{\sqrt{2}}) - (0$

$\left(\frac{1}{2} + \ln\frac{1}{\sqrt{2}}\right)$

(1, 0)

12. A population of bacteria grows at the rate of ln 2 per hour. If the population now is 1000 bacteria, after 3 hours the population will be

(a) 3000.

(b) 8000.

(c) 4000.

(d) 6000.

$\ln 2\, e^{-k}$

$y = 1000e^{-3\ln 2}$

13. If $4^x = 3^{2-x}$ then $x =$

(a) $-\frac{\ln 9}{\ln 12}$.

(b) $-\frac{\ln 3}{\ln 12}$.

(c) $\frac{\ln 9}{\ln 4}$.

(d) $\frac{\ln 9}{\ln 12}$.

$3^2 \cdot 3^{-x}$

$x\ln 4 = 2-x\ \ln 3$

$\frac{\ln 4}{\ln 3} = \frac{2-x}{x}$

$\frac{4^x}{3^{-x}} = 3^2$

$4^x \cdot 3^x = 9$

$\frac{4^x}{3^{-x}} = 9$

$\ln(12)^x = \ln 9$

$(4\times 3)^x = 9$

$x\frac{\ln 12}{\ln 12} = \frac{\ln 9}{\ln 12}$

$x\frac{\ln 12}{\ln} = \ln 9$

الحل الأكيد

14. The volume of the solid whose cross sections perpendicular to the $x-$axis are disks with diameters running from $y = -\sqrt{x}$ to $y = \sqrt{x}$, $0 \le x \le 1$ is

(a) $\frac{\pi}{2}$.
(b) π.
(c) $\frac{1}{2}$.
(d) 2π.

$A = \pi r^2$

$\int_0^1 \pi x$

$r = \frac{\sqrt{x}+\sqrt{x}}{2} = \frac{2\sqrt{x}}{2} = \sqrt{x}$

$= \frac{\pi x^2}{2}\Big|_0^1 = \frac{\pi}{2}$

15. The area of the surface generated by revolving the curve $y = e^x$, $0 \le x \le 1$ about the $x-$axis is

(a) $S = 2\pi \int\limits_1^e u\sqrt{1+u^2}du$.
(b) $S = 2\pi \int\limits_1^e u^2\sqrt{1+u^2}du$.
(c) $S = 2\pi \int\limits_1^e \sqrt{1+u}du$.
(d) $S = 2\pi \int\limits_1^e \sqrt{1+u^2}du$.

$S = \int 2\pi y \sqrt{1+\frac{dy}{dx}}\, dx$

$\frac{dy}{dx} = (e^x)^2 = (e^{2x})$

$S = \int_0^1 \boxed{2\pi} e^x \sqrt{1+e^{2x}}$

$= \int 2\pi e^x$

$U = 1 + 2e^{2x}$

$dU = 2e^{2x}dx$

$dx = \frac{dU}{2e^{2x}}$

16. One of the following is true

(a) e^x and e^{2x} grow at the same rate.
(b) x grows faster than $\ln x$.
(c) x and $\ln x$ grow at the same rate.
(d) x^{99} grows faster than 2^x.

$\lim\limits_{x\to\infty} \frac{\ln x}{x} = \frac{1}{x} = 0$

$\boxed{U = e^x}$

$dU = e^x dx$

$S = \int_1^e 2\pi \sqrt{1+u^2}\, du$

$dx = \frac{dU}{e^x}$

17. $\int\limits_0^1 e^x \cosh x\, dx =$

$e^x\left(\frac{e^x + e^{-x}}{2}\right) = \int \frac{e^{2x}+1}{2} = \frac{1}{2}\left[\frac{e^{2x}}{2} + x\right]_0^1$

$= \frac{1}{2}\left[\left(\frac{e^2}{2}+1\right) - \left(\frac{1}{2}\right)\right]$

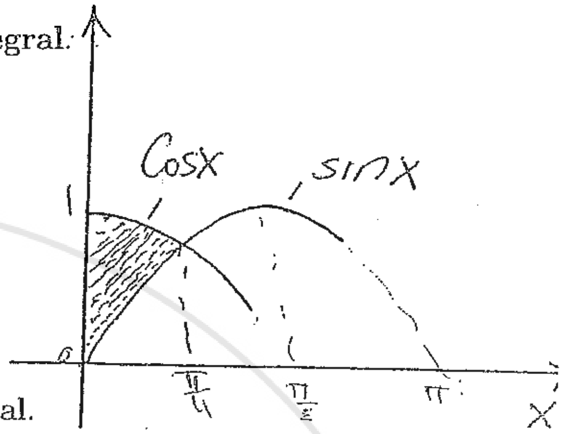
$= \frac{1}{2}\left[\frac{e^2}{2}+\frac{1}{2}\right) = \frac{e^2+1}{2}\cdot\frac{1}{2}$

$\frac{e^2+1}{4}$

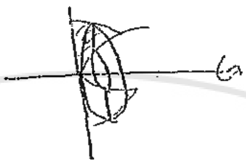
(a) $\frac{e^2-3}{4}$.
(b) $\frac{e^2+1}{4}$.
(c) e^2+1.
(d) $\frac{e^2}{4}$.

Question 2(16%) Consider the area enclosed between the curves $y = \sin x$, $y = \cos x$ and the y-axis. Setup integrals that give the volume of the solid generated by revolving this area about

(i) The x-axis. Use washer method. **Don't solve the integral.**



$$V = \pi \int_0^{\frac{\pi}{4}} \cos^2 x - \sin^2 x \; dx$$



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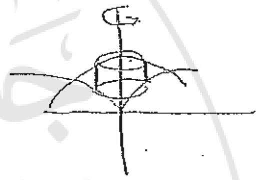
(ii) The y-axis. Use shell method. **Don't solve the integral.**

$$V = \int_0^{\frac{\pi}{4}} 2\pi (x)(\cos x - \sin x)\, dx$$



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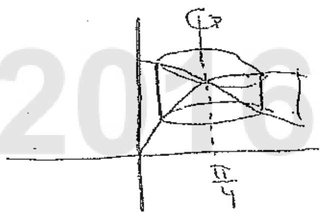
(iii) The line $x = \frac{\pi}{4}$. Use shell method. **Don't solve the integral.**

$$V = \int_0^{\frac{\pi}{4}} 2\pi \left(\frac{\pi}{4} - x\right)(x) \; dx$$



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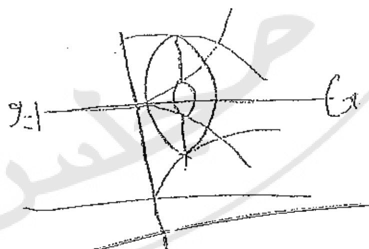
(iv) The line $y = 1$. Use washer method. **Don't solve the integral.**

$$V = \pi \int_0^{\frac{\pi}{4}} (1 - \sin x)^2 - \cos^2 x \; dx$$



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$\cos x = \sin x$

$\frac{\sin x}{\cos x} = 1$

$\frac{\pi}{4}$

14

Question 3(16%) Solve the following integrals:

(a) $\int \sin^{-1} x\, dx$.

$U = \sin^{-1} x \qquad dV = dx$

$dU = \frac{dx}{\sqrt{1-x^2}} \qquad V = X$

$= X \sin^{-1} X - \int \frac{X}{\sqrt{1-x^2}} dx$

+8

$\int \frac{X}{\sqrt{1-x^2}} dX = \int \frac{\cancel{X}}{\sqrt{U}} \cdot \frac{dU}{-2\cancel{X}} = -\frac{1}{2} \int \frac{dU}{\sqrt{U}} = -\frac{1}{2} \int U^{-\frac{1}{2}} dU$

$U = 1 - x^2$

$dU = -2X\, dX$

$dX = \frac{dU}{-2X}$

$= -\frac{1}{2} \cdot 2 U^{\frac{1}{2}} = -\sqrt{1-x^2}$

$\int \sin^{-1} X\, dX = X \sin^{-1} X + \sqrt{1-X^2} + C$

(b) $\int \frac{(x-1)dx}{(x+1)(x^2+1)}$. $= \frac{A}{X+1} + \frac{BX+C}{X^2+1}$

$(X-1) = A(X^2+1) + (BX+C)(X+1)$

$(X-1) = AX^2 + A + BX^2 + BX + CX + C$

+8

$X-1 = (A+B)X^2 + (B+C)X + A + C$

$0 = A + B \to (1) \longrightarrow A = -B$

$1 = B + C \to (2)$

$-1 = A + C \to (3) \longrightarrow -1 = -B + C$

$1 = B + C$

$\boxed{C = 0}$ $\boxed{B = 1}$ $\boxed{A = -1}$

$\int \frac{X-1}{(X+1)(X^2+1)} dX = \int \frac{-1}{X+1} dX + \int \frac{X}{X^2+1} dX$

$= -\ln|X+1| + \frac{1}{2} \ln|X^2+1| + C$

16

17

Question 4(17%) Consider the curve $y = \ln x$, $1 \le x \le \sqrt{3}$.

(a) Show that the length of the curve $L = \int_1^{\sqrt{3}} \frac{\sqrt{1+x^2}}{x} dx$.

$L = \int_1^{\sqrt{3}} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$

$\frac{dy}{dx} = \frac{1}{x} dx \rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{1}{x^2} \rightarrow \sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{1+\frac{1}{x^2}}$

$\int = \int_1^{\sqrt{3}} \sqrt{\frac{1}{1}+\frac{1}{x^2}}\, dx = \int_1^{\sqrt{3}} \sqrt{\frac{x^2+1}{x^2}}\, dx = \int_1^{\sqrt{3}} \frac{\sqrt{1+x^2}}{\sqrt{x^2}}\, dx = \int_1^{\sqrt{3}} \frac{\sqrt{1+x^2}}{x}\, dx$

(b) Solve the integra in (a).

$\int_1^{\sqrt{3}} \frac{\sqrt{1+x^2}}{x}$, $x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

$= \int_1^{\sqrt{3}} \frac{\sqrt{1+\tan^2\theta}}{\tan\theta} \sec^2\theta\, d\theta = \int_1^{\sqrt{3}} \frac{\sec^3\theta}{\tan\theta} d\theta$

$u = \sqrt{1+x^2}$

$du = 2x\, dx$

$dx = \frac{du}{2x}$

$\sqrt{1+x^2}$, x, 1

$\int_1^{\sqrt{3}} \sqrt{\frac{1+x^2}{x^2}} = \int_1^{\sqrt{3}} \frac{\sqrt{1+x^2}}{x} dx = \int \frac{u^2}{x^2} du$

$u = \sqrt{1+x^2}$

$du = \frac{2x}{2\sqrt{1+x^2}} dx$

$dx = \frac{\sqrt{1+x^2}}{x}$

$= \int \frac{u^2}{u^2-1} du$

$u^2 - 1 \overline{)\, u^2}$
$\quad -u^2 \pm 1$
$\quad 1$

$u^2 = 1 + x^2$

$u^2 - 1 = x^2$

$= \int 1\, du + \int \frac{1}{u^2-1} du$ تحليل كسور

$\frac{1}{u^2-1} = \frac{A}{u-1} + \frac{B}{u+1} \Rightarrow A(u+1) + B(u-1)$

$u = \tan\theta\, d\theta$

$du = \sec^2\theta\, d\theta$

$d\theta =$

$u = \sqrt{1+x^2}$

$du = \frac{x}{\sqrt{1+x^2}}$

$\frac{\sec \cdot \sec^2}{\tan\theta}$

$dx = \frac{\sqrt{1+x^2}}{x} dx$

$\sqrt{1+\frac{1}{x^2}}$

$\sqrt{u-1}$

$u - 1 = \frac{1}{x^2}$

$u = 1 + \frac{1}{x^2}$

$\frac{1}{\cos\theta}$ $\frac{\cos\theta}{\sin\theta}$

$du = \frac{-1}{x} dx$

$\frac{1}{\cos\theta}$ $\frac{1}{\sin\theta}$

$dx =$

$dx = -du\, x$

$1 = Au + A + Bu - B$

$1 = (A+B)u + (A-B)$